Unit VI: Electricity

Coulomb's Law

Remember electrostatics:

Like charges repel.

Opposite charges attract.

Charge is measured in Coulombs (C). It is named in honour of Charles Augustin de Coulomb.

Common subatomic particles can have a charge, as summarized in this table:

Particle	Charge
Electron (e^-)	-1.60×10^{-19} C
Proton (p^+)	$+1.60 \times 10^{-19}$ C
Neutron (n^0)	0 C

A charge of 1.60×10^{-19} C is called an elementary charge and has a symbol of "e". Do not confuse this with the symbol for electron, as there is no negative sign on the symbol.

One C is equivalent to the charge on 6.25×10^{18} particles.

Ex. What is the charge on 12 protons?

$n = 12$
$e = 1.60 \times 10^{-19} C$
$q = ?$

$q = ne$
$q = 12(1.60 \times 10^{-19} C)$

$\boxed{q = 1.92 \times 10^{-18} C}$

$q = ne$	Where q = charge (C) n= number of elementary particles e = 1.60×10^{-19} C

Coulomb found that the force between charges is dependent on the charge of the objects and their distance apart.

$\lvert F_e \rvert = \frac{kq_1q_2}{r^2}$	Where Fe = force (N) q = charge (C) r = distance between the charges (m) k = 8.99×10^9 Nm^2/C^2

Coulomb's Law will just give us the magnitude of the electric force. We will use information about the charge on the objects to determine the direction. The positive and negative values from the formula above do not come from direction, but from positive and negative charges.

$\times 10^{-6}$ $\mu = \times 10^{-6}$ $\times 10^{-6}$

Ex. A comb with -2.0 µC of charge is 0.15 m to the left from a hair with 3.0 µC of charge. Determine the force the hair exerts on the comb.

$q_c = -2.0\times10^{-6}C$

$r = 0.15m$

$q_h = 3.0\times10^{-6}C$

$F_e = ?$

E → S

$F_e = \frac{kq_cq_h}{r^2}$

$F_e = \frac{8.99\times10^9(-2.0\times10^{-6}C)(3.0\times10^{-6}C)}{(0.15m)^2}$

$F_e \simeq -2.4N$

Attractive

$\vec{F_e} \simeq 2.4N$ [Right]

In the question above, the charges were in micro coulombs. Charges that are seen in day to day life tend to be about 1 µC. Things like a lightning bolt may have a charge of 1 or 2 C.

You may also notice that Coulomb's Law is very similar to Newton's Law of Universal Gravitational. What is one significant difference between the two?

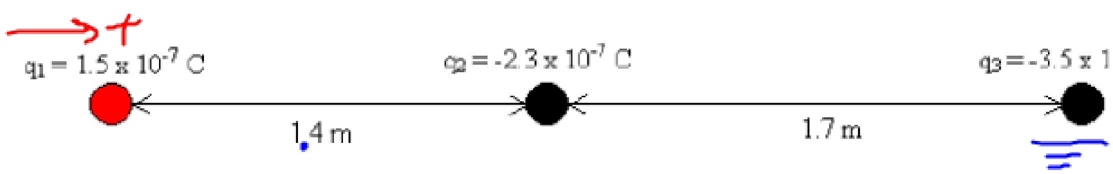
G is very small, but k is huge!

What does this say about the strength of the gravitational force versus that of the electrostatic force?

Gravity is weak, electrostatic strong!

Another important difference is that the gravitational force can only cause an attraction, but the electrostatic force can cause attraction or repulsion.

Ex. The following three charges are arranged as shown. Determine the net force acting on the far right.



① Force of q_1 on q_3

$_1F_3 = \frac{kq_1q_3}{r^2}$ (F_e)

$_1F_3 = \frac{8.99\times10^9(1.5\times10^{-7}C)(-3.5\times10^{-4}C)}{(1.4m+1.7m)^2}$

$_1F_3 = -0.049112903N$

$_1\vec{F_3} = -0.0491N$

② Force of q_2 on q_3

$_2F_3 = 0.25041349 5N$

$_2F_3 \simeq 0.2504N$

③ Find F_{net}

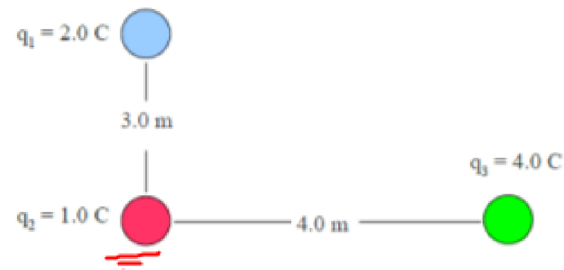
$\vec{F}_{net} = {}_1\vec{F}_3 + {}_2\vec{F}_3$

$= -0.0491N + 0.2504N$

$= 0.2504N$

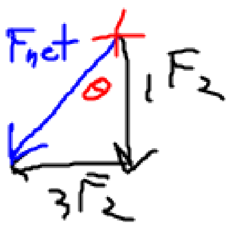
$\boxed{F_{net} \simeq 0.25N [Right]}$

Ex. Three charges are arranged in a right angle triangle as the diagram shows. Determine the force on q_2.



$${}_1F_2 = \frac{kq_1q_2}{r^2} = \frac{8.99\times10^9(2.0)(1.0)}{(3.0)^2} = 1997777777N$$

$${}_3F_2 = \frac{kq_2q_3}{r^2} = \frac{8.99\times10^9(1.0)(4.0)}{(4.0m)^2} = 2247500000N$$



Pythagorus:

$$F_{net} = \sqrt{(1997777777N)^2 + (2247500000N)^2}$$

$$F_{net} \simeq 3.0\times10^9 N$$

$$\theta = \arctan\left(\frac{{}_3F_2}{{}_1F_2}\right)$$

$$\theta = \arctan\left(\frac{2247500000N}{1997777777N}\right)$$

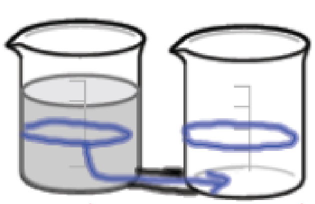
$\theta \simeq 48°$

$270° - 48° = 222°$

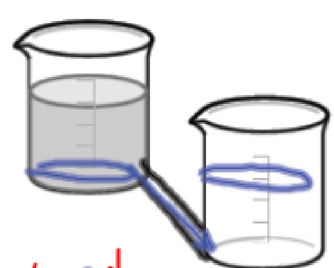
$$\boxed{\vec{F}_{net} = 3.0\times10^9 N [222°]}$$

Flow of Charge

Water Analogy:



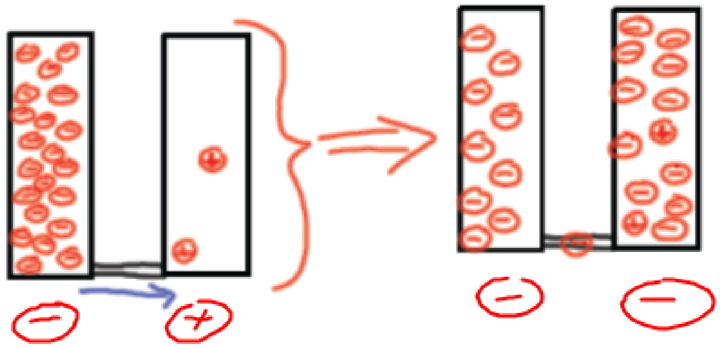
Water flows because the potential energy differences that exist between the tops of the water.



- water will flow until at equal levels

- water will flow until top of water at equal levels

Charge flows due to a potential difference in charge between the two charged objects. The charge will flow until it is in equilibrium.



When a conductor is connected between both terminals of a battery, it forms an electric circuit. The battery in the circuit will cause charge to flow from one terminal to another. This is called electric current.

A better definition of current is that it is the amount of charge that passes a given point in a certain amount of time.

$I_{avg} = \frac{q}{\Delta t}$	Where I_{avg} = current in amperes (A) q = charge in coulombs (C) Δt = time in seconds (s)

Ex. A current of 1.42 A flows through a wire connecting the terminals of a battery. After 4.00 minutes, how much charge has passed through the circuit?

$I = 1.42\ A$
$\Delta t = 4.00\,min = 240\,s$
$q = ?$

$I = \frac{q}{\Delta t}$

$I\Delta t = q$

$q = 1.42A(240\,s)$

$q = 340.8\ C$

$\boxed{q \approx 341\ C}$

Remember, it is the free electrons in a conductor that can move around. And while we talk about the electrons "flowing", they are really only wiggling back and forth.

Voltage

Voltage is sometimes referred to as the electric potential difference, the electric potential, or potential difference. The electric potential difference is the reason that current flows – without it, nothing would happen. It is the force that pushes the electrons. Voltage is the electric potential energy per unit charge. It is how much work is needed per Coulomb of charge. If something has more charge, it needs more work to move it.

The Volt (V) is the derived until named in honour of Alessandro Volta. One V is equivalent to a J/C.

$V = \frac{W}{q} = \frac{\Delta E}{q}$	Where V = voltage (V) W = ΔE = electric potential energy (J) q = charge (C)

Ex. How much voltage is needed to do 2500 J of work moving 3.5×10^{-3} C?

$v = ?$
$W = 2500\,J$
$q = 3.5\times10^{-3}\,C$

$$V = \frac{W}{q}$$

$$V = \frac{2500\,J}{3.5\times10^{-3}\,C}$$

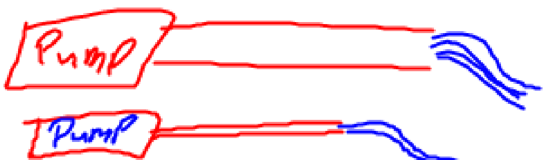
$$V = 714285.714285\,V$$

$$\boxed{V \approx 7.1\times10^{5}\,V} \quad \text{or } 0.71\,MV$$

Resistance

The amount of current flowing in a circuit depends partly upon the voltage. If you increase the voltage, you increase the "pumping power" that is moving current through the circuit. Current flow also depends on how the material of the conductor resists the motion of electrons.

Often, a water pump is used as an analogy for electricity.

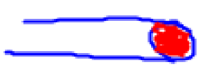


The pump is like voltage. It is the pumping power that is shoving electrons through the wire.

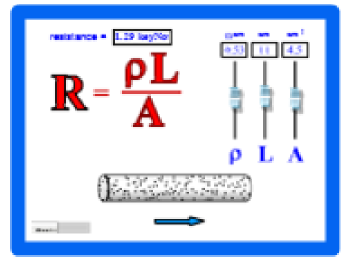
The hose diameter is like the wire's resistance. A thin hose lets only a little water through, just like a poor conductor lets only a little current through.

The water is like current. A little water pouring out is like a little current running through a wire.

Resistance depends upon four factors:

Property	Relationship	Example
Resistivity of material	Different materials used as conductors result in different resistances. This is compared by assigning a resistivity (ρ) to materials.	Gold has a low resistivity because it is such a good conductor.
Cross section area of wire 	Larger area results in low resistances.	A large diameter wire is a wider opening for electrons to move through, just like a wide doorway can allow more people to walk easily through.
Length of wire	Shorter lengths of wire result in low resistances.	Moving current through a long piece of wire is like forcing the electrons to run a marathon race. A lot of energy is used up just trying to get to the end.
Temperature	Higher temperatures result in higher resistances.	When conductors are colder, their atoms move less. This makes it easier for electrons to "move through".

We will not take into account the temperature coefficient (α) in our calculations.



ρ

$R = \frac{\rho l}{A}$	Where R = resistance (ohms = Ω) ρ = resistivity (Ωm) l = length (m) A = area (m^2)

Ex. Wire gauge is a method used to compare different cross sectional area of wire; the higher the gauge, the smaller the area. 14-gauge wire has a diameter of 1.628 mm. If aluminum has a resistivity of 2.82×10^{-8} Ωm, determine the resistance of a 12.5 m long piece of aluminum wire.

$d = 1.628 m$
$= 1.628 \times 10^{-3} m$
$\rho = 2.82 \times 10^{-8} \Omega m$
$L = 12.5 m$
$R = ?$

$R = \frac{\rho L}{A}$

$R = \frac{\rho L}{\pi (\frac{d}{2})^2}$

$R = \frac{2.82 \times 10^{-8} \Omega m (12.5 m)}{\pi \left(\frac{1.628 \times 10^{-3} m}{2}\right)^2}$

$\boxed{R \simeq 0.169 \Omega}$

Ohm's Law

Georg Simon Ohm made a discovery about certain conductors. If a conductor's resistance stays constant even when different voltages are applied to it, the conductor is said to obey Ohm's Law. For our course, all conductors will obey Ohm's Law.

V = IR	Where V = voltage (V) I = current (A) R = resistance (Ω)

Ex. A 30Ω resistor is hooked up to a 12 V battery. How much current flows?

$R_1 = 30\Omega$

12V

$V = 12 V$
$\Omega = 30\Omega$
$I = ?$

$V = IR$

$\frac{V}{R} = I$

$\frac{12 V}{30 \Omega} = I$

$\boxed{I = 0.4 A}$

Transfer of Energy

Like other forms of energy, electricity can change forms. Combining equations from this chapter and from work and energy, we get the following:

$V = IR \Rightarrow I = \frac{V}{R}$

$P = \frac{qV}{t}$	$P = VI$
$P = \frac{V^2}{R}$	$P = I^2R$

These will be the main formulas we will use when calculating power in an electrical circuit.

Ex. A household appliance draws a 0.50 A current. Determine the power rating of this appliance.

$I = 0.50A$
$P = ?$
$V = 120V$

$P = VI$

$P = 120V(0.50A)$

$\boxed{P = 60.\ W}$

Ex. A portable fridge in the back of a van has a power rating of 5.0 W. Since it is plugged into the van's electrical system, it runs off of 12 V. Determine the resistance of the fridge.

$P = 5.0W$
$V = 12.0V$
$R = ?$

$P = \frac{V^2}{R}$

$R = \frac{V^2}{P}$

$R = \frac{(12.0V)^2}{5.0W}$

$R = 29\Omega$

We can also combine a number of equations to get relationship for electrical energy.

E=PΔt	E=VIΔt	E=I^2RΔt

Ex. A heater has a resistance of 10.0 Ω. It operates on 120.0 V.

What is the current through the resistance?

$R = 10.0\Omega$
$V = 120.0V$
$I = ?$

$V = IR$

$\frac{V}{R} = I$

$I = \frac{120.0V}{10.0\Omega}$

$\boxed{I = 12.0A}$

What thermal energy in joules is supplied by the heater in 10.0 s?

$E = ?$
$I = 12.0A$
$V = 120.0V$
$R = 10.0\Omega$
$t = 10.0s$

$E = VI\Delta t$

$E = 120.0V(12.0A)(10.0s)$

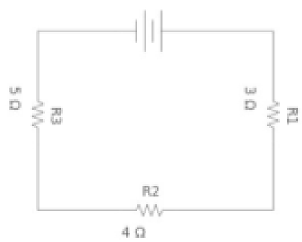
$\boxed{E = 14400J}$

Circuits

Any path along which electrons can flow is a circuit. For a continuous flow of electrons, there must be a complete circuit with no gaps. Circuits can be broken down into two main categories: series and parallel.

In any circuit, going through a battery increases voltage and going across a resistor decreased voltage (a voltage drop). As you go around any single pathway, they total voltage drops across all resistors must equal the voltage from the battery.

Series Circuits



In a series circuit, there is only one path for the electrons to flow. The current is constant everywhere in a series circuit. If you break the circuit at any point, the entire circuit will stop working.

The current has to flow through all the resistors, so we increase the resistance of the circuit as we add more resistors. Since the total drops in voltage must equal the voltage from the battery along a single path, we can say:

$$V_T = V_1 + V_2 + V_3 + \cdots$$

Since V = IR, we can also say:

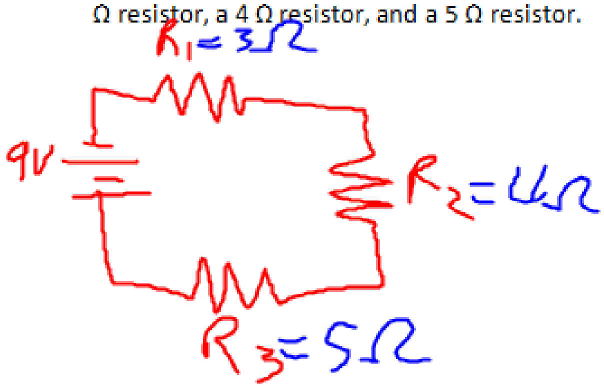
$$IR_T = IR_1 + IR_2 + IR_3 + \cdots$$

Because the current is the same anywhere in a series circuit, we can divide both side by I and get:

$$R_T = R_1 + R_2 + R_3 + \cdots$$

The 'T' subscript refers to the total voltages and resistances of the entire circuit. This means that if we take all the resistances and add them up, we get a single equivalent resistance. At that point, you can use Ohm's Law to find the actual current going through the circuit.

Ex. Determine the voltage drops and current flowing through each part of a circuit containing a 9 V battery, a 3 Ω resistor, a 4 Ω resistor, and a 5 Ω resistor.



	V	I	R
R_1			3Ω
R_2			4Ω
R_3			5Ω
R_T	9V		

Find equivalent resistance:

$R_T = R_1 + R_2 + R_3$

$R_T = 3\Omega + 4\Omega + 5\Omega$

$R_T = 12\Omega$

	V	I	R
R_1			3
R_2			4
R_3			5
R_T	9		12Ω

Use Ohm's Law to find current:

$V_T = I_T R_T$

$I_T = \frac{V_T}{R_T}$

$I_T = \frac{9V}{12\Omega}$

$I_T \approx 0.75A$

	V	I	R
R_1			3
R_2			4
R_3			5
R_T	9	0.75A	12

The current is the same everywhere, so we can use that value of I for all the resistors.

$I_T = I_1 = I_2 = I_3$

	V	I	R
R_1		0.75A	3
R_2		0.75A	4
R_3		0.75A	5
R_T	9	0.75	12

Use Ohm's Law to find the voltage drop of each resistor:

R_1

$V = IR_1$

$V = 0.75A(3\Omega)$

$V_1 = 2.25\ V$

R_2

$V = 0.75A(4\Omega)$

$V_2 = 3V$

R_3

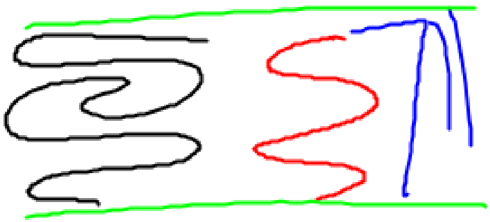
$V = 0.75A(5\Omega)$

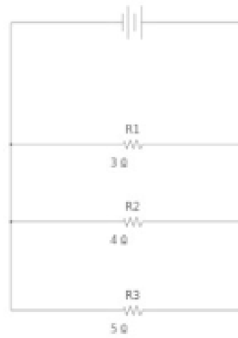
$V_3 = 3.75V$

	V	I	R
R_1	2.25V	0.75	3
R_2	3V	0.75	4
R_3	3.75V	0.75	5
R_T	9	0.75	12

You can check your work by making sure that the sum of the voltage drops across the three resistors equals the same voltage as your power source.

Parallel Circuits





In a parallel circuit, there is more than one pathway for the electron flow between terminals. Since each resistor is in direct connection to both terminals of the battery, each resistor has the same full voltage from the battery in a parallel circuit.

As long as it isn't on the main branch, a break anywhere in the circuit does not affect the other resistors if they still have a path to the battery.

Because charge is conserved, the current flowing into a junction must equal the current flowing out. Similar to what we did with series circuits, we get the following:

$$I_T = I_1 + I_2 + I_3 + \cdots$$

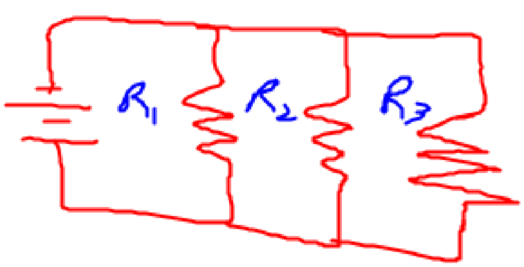
$V = IR \Rightarrow I = \frac{V}{R}$

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots$$

Because in a parallel circuit the voltage is the same among all the resistors, we can divide out V:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Ex. Determine the voltage drops and current flowing through each part of a circuit containing a 9 V battery, a 3 Ω resistor, a 4 Ω resistor, and a 5 Ω resistor if the resistors are all wired in parallel.



	V	I	R
R_1			3Ω
R_2			4Ω
R_3			5Ω
R_T	9V		

Scrunch the parallel resistors down to one equivalent resistor:

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$\frac{1}{R_T} = \frac{1}{3\Omega} + \frac{1}{4\Omega} + \frac{1}{5\Omega}$

$\left(\frac{1}{R_T}\right)^{-1} = (0.783333)^{-1}$

$R_T \simeq 1.2765957$

Calculate the current using the equivalent circuit:

$V = IR$

$I = \frac{V}{R}$ $I = 7.05A$

$I = \frac{9V}{1.2765957\Omega}$

	V	I	R
R_1			3
R_2			4
R_3			5
R_T	9	7.05A	1.2765957Ω

The voltage is the same everywhere so we can unscrunch the circuit and put in the voltage for each resistor:

$V_T = V_1 = V_2 = V_3 = 9V$

	V	I	R
R_1	9V		3
R_2	9V		4
R_3	9V		5
R_T	9	7.05	1.276595745

Now we can solve for the missing currents in each of the individual branches of each resistor:

$V = IR$

$I = \frac{V}{R}$

R_1

$I_1 = \frac{9V}{3\Omega}$

$I_1 = 3A$

R_2

$I_2 = \frac{9V}{4\Omega}$

$I_2 = 2.25A$

R_3

$I_3 = \frac{9V}{5\Omega} = 1.8A$

	V	I	R
R_1	9	3A	3
R_2	9	2.25A	4
R_3	9	1.8A	5
R_T	9	7.05	1.276595745

To check your work, add all of the currents going through the separate branches. It should add up to the current on the main branch.

Kirchoff's Laws

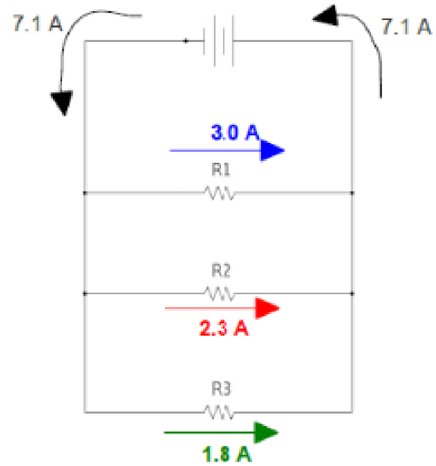
Sometimes, we encounter a circuit that is too complicated for simple analysis. This may be because the circuit is a mix of series and parallel, or has more than one power source. G. R. Kirchoff invented some rules to deal with these cases. The rules boil down to convenient applications of the laws of conservation of charge and energy.

In the examples above, you were already sort of using his rules, unbeknownst to you.

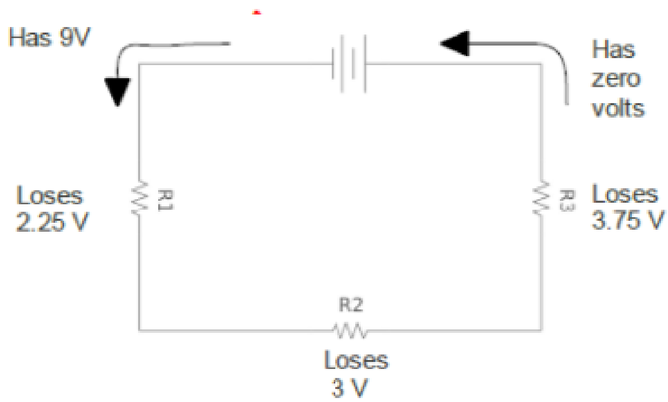
The First Rule : The Junction Rule:

"At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction."

This means that when current reaches the branches in a parallel circuit, it will split up and take different routers. When the branches come back together, the currents will add back together too.



The Second Rule: The Loop Rule



"The algebraic sum of the changes in potential around any closed path of a circuit must be zero."

This rule is based on the conservation of energy. This just means that when you look at resistors in series, the drop in voltage across all of them will be equal to whatever the source is.

These two rules become important when you need to analyze combination circuits.

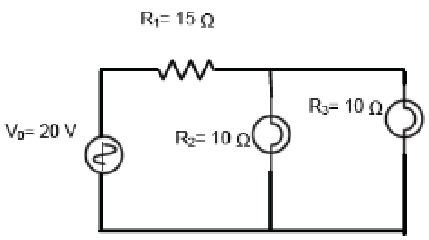
Combination Circuits

Combination circuits are more realistic because they are circuits with series and parallel parts together. You'll recognize them when you see at least a couple of resistors are in parallel, but after you scrunch them, you end up with a series circuit for the other resistors.

To solve circuits that are a combination of series and parallel, flip back and forth through Kirchoff's Laws as appropriate. Use Ohm's Law any time you know two things at one point.

As a general rule, try to scrunch the parallel parts first. Once that is done, scrunch the series parts. Skipping steps leads to errors. You might have to scrunch resistors in series within a parallel circuit first.

Ex. Solve the following circuit.



	V	I	R
R_1	15V ④	1A ④	15Ω
R_2	5V ⑤	0.5A ⑥	10Ω
R_3	5V ⑤	0.5A ⑥	10Ω
Total	20V	1A ③	20Ω ②

① Scrunch Parallel

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{23}} = \frac{1}{10\Omega} + \frac{1}{10\Omega}$$

$$\frac{1}{R_{23}} = \frac{1}{5\Omega}$$

$$R_{23} = 5\Omega$$

② Scrunch Series

$$R_T = R_1 + R_{23}$$

$$R_T = 15\Omega + 5\Omega$$

$$R_T = 20\Omega$$

③ Find I_T

$$V_T = I_T R_T$$

$$\frac{V_T}{R_T} = I_T$$

$$I_T = \frac{20V}{20\Omega}$$

$$I_T = 1A$$

④ Solve R_1

Series: $I_T = I_1 = 1A$

$$V_1 = I_1 R_1$$

$$V_1 = 1A(15\Omega)$$

$$V_1 = 15V$$

⑤ Find voltage of parallel part

$$V_T = V_1 + V_{23} \leftarrow \text{Parallel}$$

$$V_T - V_1 = V_{23}$$

$$20V - 15V = V_{23}$$

$$5V = V_{23}$$

⑥ Find I_2 and I_3

$$V = IR \qquad I_2 = \frac{5V}{10\Omega} = 0.5A$$

$$I = \frac{V}{R} \qquad I_3 = \frac{5V}{10\Omega} = 0.5A$$